

Reliability Markov Chains for Security Data Transmitter Analysis

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Abstract

Security communication systems composed of highly reliable components may have few if any failures while undergoing heavy testing or field-usage. This paper combines (i) analysis of randomized pulse modulation schemes based on finite Markov chains with (ii) security communication systems failure as a rare event with a finite-state, discrete-parameter, recurrent Markov chain that models the failures. An application of these results would be (i) in the analysis of randomized pulse width modulation implemented into the command of the railway data transmitter for the traffic security, and (ii) in the analysis of these system's reliability.

1. Introduction

Security communication reliability model holds observed success/failure data or estimates the component reliability within the framework of probability models in order to predict patterns of future performance. The probability distributions for the number of failures and the failure occurrence within a specified time (e.g., mean time to failure: MTTF) are often adapted from hardware reliability theory [1] or justified empirically [2]. By failure as a rare event [3] we mean that its probability of occurrence is greater than zero but smaller by at least several orders of magnitude than non-failure events in security data communications. In this situation, the MTTF is a large number. Thus, security data communications which receive heavy usage according to established usage distributions are a candidate for the treatment of their failures as rare events. Intuitive definitions and convenient computations are attributes of the Markov chain for work with rare events. The Markov chain provides not only a convenient definition of a rare event, i.e., a visit to an abnormal fail-state, but also a direct measure of rarity using steady-states

probabilities. In our approach, a discrete-parameter, finite-state Markov chain [5] is used to represent both security data communications failures (as transitions to a rare fail-state) and randomized pulse width modulation schemes of data transmitter.

This approach has a few advantages: the randomized modulation of switching in power converters reduces filtering equipments and allows an explicit control in time domain performance. Randomized modulation is very effective for narrow band constraints [6]; therefore it is proper to be implemented into the command of the railway data transmitter for the traffic security. This paper is organized as follows: Section 2 describes the working principles of security railway data transmitter CN-75-6. Section 3 deals with the Markov chains formalisms of synthesis in randomized modulation. Section 4 gives an example of randomized modulation governed by periodic Markov chains, and introduces a new structure of complex periodic Markov chain. Section 5 estimates the reliability of this approach, with an example, by framing it into a model, respectively the reliability Markov chain model (RMCM). Conclusions synthesize the results obtained, both for randomized modulation Markov model and for its reliability estimation.

2. Security railway data transmitter

Typically, a data transmitter consists of a logic command and a switching power converter. The switching power converter uses only switching devices, energy storage elements, and relays an appropriate modulation of the switches to convert the available a.c. or d.c. voltage (current) waveforms of the power source into the a.c. or d.c. wave forms required. The switches are generally semiconductor devices: diodes, thyristors, IGBT, etc. A switching scheme involves generating a switching function $f(t)$, which by definition has the volume one when the switch is conducting and the zero value otherwise. This is schematically indicated in figure 1.

Since power converters generally operate in a periodic steady state, transmitter wave forms of interest (in the electronic code transmitter CN-75-6) are typically periodic functions of time, in the steady state, as illustrated in figure 2. The structure of the codified pulses of the CN-75-6 transmitter involves 10 sine waves of 75 Hz frequency followed by a pause virtually carrying a multiple of 10 sine waves of 75 Hz frequency. Because the information is carried by pauses, the CN-75-6 decoder counts the number of sine waves periods which fits the length of the pause.

The resulted number, which must be a multiple of the sinusoids, indicates a specific command for the railway installations, such as a color displayed by the traffic lights. As shown in figure 2 the switching function $f(t)$ determines the structure of the codified pulses $g(t)$ of the CN-75-6 transmitter [4]. The function $f(t)$ given by the controller (figure 1) reflects steady-state wave forms for the controlled voltages, respectively for the controlled code.

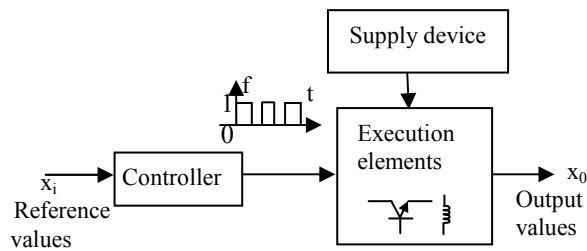


Figure 1. Schematic representation of a switching transmitter

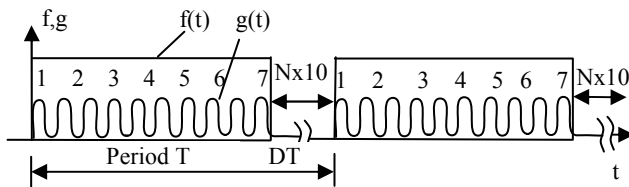


Figure 2. Switching functions of the electronic code transmitter CN-75-6

It is well known that the average value or duty ratio D , of $f(t)$ usually determines the nominal output of a d.c./d.c. converter, while the fundamental component of $f(t)$ usually determines the output of a d.c./a.c. converter; similar statements can be made for a.c./d.c. or a.c./a.c. converters.

Converters wave forms that are periodic have spectral components only at integer multiples of the fundamental frequency.

The allowable harmonic content of some of these waveforms is constrained: our circuit, type CN-75-6, ideally should have only the 75Hz fundamental component present. In this case, stringent filtering requirements may be imposed to the railway insulation resistance, and on the power converter. As the use of pulse width modulation (PWM) in power converters controlled by microprocessors evolved, new methods became available to address the effects of electromagnetic interference.

While an effort was directed toward the optimization of deterministic PWM waveforms, an alternative in the form of randomized modulation for d.c./a.c. and a.c./a.c. conversion is based on schemes in which successive randomizations of the periodic segments of the switching pulse train are statistically independent and governed by probabilistic rules. These schemes are denoted as stationary [3], [19].

In this paper we describe an approach to the synthesis of this class of stationary randomized modulation schemes that enables explicit control of the time-domain performance of the CN-75-6 railway data transmitter, used in the Romanian railways system.

We notice that our approach can easily be implemented to a large class of data transmitters with PWM waveforms, using stationary randomized modulation for increasing the security of transmissions.

3. Switching synthesis with Markov chains

The basic synthesis in randomized modulation is to design a randomized switching procedure that minimizes given criteria for spectral characteristic of $f(t)$, while respecting time-domain behavior constraints. Practically, optimization procedure assumes the minimization of discrete spectral components (denoted as narrow-band optimization), and the minimization of signal power in a given frequency range (denoted as wide-band optimization, [5]). The case of pulse trains specified by periodic Markov chains is denoted as ergodic cyclic [4], [5]. We assume that the state of the chain goes through a sequence of n classes of states C_i , occupying a state in each class for an average time σ_i , $i = 1, \dots, n$. The time-average autocorrelation [6] of a random process $f(t)$ is defined as:

$$R_f(\tau) = \lim_{n \rightarrow \infty} \frac{1}{2w} \int_{-w}^w E[f(t) \cdot f(\tau + t)] \cdot dt \quad (1)$$

Where the expectation $E[.]$ refers to the whole ensemble $[.]$. The contribution that states the Markov chain belonging to the class C_k , with the time-averaged

autocorrelation (1) is scaled by $\tau_k / \sum_{i=1}^n \tau_i$, where τ_i is

the expected time spent in the class C_i before a transition into the class C_{i+1} .

We define P as the $n \times n$ state-transition matrix, and its (k, i) th entry is the probability that at the next transition the chain goes to state i , given that it is currently in state k .

Each row of P sums to 1; P is thus a stochastic matrix, and therefore has a single eigenvalue $\lambda_i = 1$, with corresponding eigenvector $1_n = [1 \ 1 \ \dots \ 1]$, and all other eigenvalues with module strictly less than one.

It can be shown [1], [17] that after a possible remembering of the states, the matrix P for a periodic Markov chain can be written in a block-cyclic form.

$$P = \begin{bmatrix} 0 & P_{12} & 0 & \dots & 0 \\ 0 & 0 & P_{23} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & P_{n-1,n} \\ P_{n1} & 0 & 0 & \dots & 0 \end{bmatrix}$$

Let P_p denote the product of sub matrices of P : $P_p = P_{n1} \cdot \dots \cdot P_{23} \cdot P_{12}$, and let v_i denote the vector of steady-state probabilities, conditional on the system being in class C_i . Then, we have:

$$V_i^* = v_i \cdot P_p \quad (2)$$

The average time spent in class C_i is:

$$\tau_i = \sum_k V_k^* \cdot \tau_k \quad (3)$$

We notice that in relation (3) the summation refers to all states in class C_i . Let $T_o = \sum_{i=1}^n \tau_i$ and let $\Theta_i = \text{diag}$

(V_i^*) . If the first pulse belongs to the class i , then the pulse $\tau_i + \tau$ belongs to the class $(i + m) / \text{mod } n$, where m represents the number of transitions between pulses τ_i and $\tau_i + \tau$.

We mention that the average duration of some classes may be null, which means that these classes (with corresponding states) are skipped [13-15].

This approach allows us to build a simplified Markov chain model (we present this assumption, which we believe to be novel, in the next section).

When we add the contribution of all classes to the average power spectrum (scaled by the relative average duration of each class), the result can be written as follows [4]:

$$S(f) = \frac{1}{T_0} \left[\sum_{i=1}^n \frac{\tau_i}{T_0} U_i^T(f) \cdot \Theta U_i(f) + 2R_i^T(f) \cdot S_C \cdot 1_n \right] + \frac{1}{T_0^2} R_i^T(f) \cdot S_C \cdot 1_n \sum_{i=-\infty}^{\infty} \left(f \frac{i}{T^*} \right) \quad (4)$$

Where T^* is the greatest common divisor of all waveform duration, 1_n is an $n \times 1$ vector of 1 and U_i is the vector of Fourier transforms of waveforms assigned to states in class C_i .

The matrix S_C has a Toeplitz structure, with (k, i) th entry:

$$S_{ck,i}(f) = \frac{\tau_i}{T_0} \cdot U_k^T(f) \cdot (I - \Lambda_k(f))^{-1} \cdot \Lambda_{k,i}(f) \cdot U_i(f) \quad (5)$$

Where Λ_k is a product of n matrices:

$\Lambda_k = Q_{k-1,k}, \dots, Q_{k,k+1}$, and $\Lambda_{k,j} = Q_{k,k+1}, \dots, Q_{i-1,i}$.

Where Q is a matrix $n \times n$ whose (k, i) entry is $Q_{k,i}(\sigma) = P_{k,i} \delta(\sigma - \tau_k)$.

Also, the (k, i) th entry of S_d is given by relation:

$$S_{dk,i}(f) = \frac{\tau_i}{T_0} \cdot U_k^T(f) \cdot V_k \cdot (V_i)^T \cdot U_i(f) \quad (6)$$

Analogous with the notion of truncated Markov chains with absorbing states [6], we propose a simplified model of Markov chains for random modulation.

The proposed Markov chain $X^{(m)}$ has the states $\{m, m+1, \dots\}$ aggregated into an absorbing class of states, which has transition matrix ${}_{(m)}T$ satisfying:

$${}_{(m)}T = \begin{bmatrix} {}_{(m)}P & p_m \\ 0 & I \end{bmatrix} \quad (7)$$

Where $p_m = \left[\sum_{j \geq m} p_{1j}, \sum_{j \geq m} p_{2j}, \dots, \sum_{j \geq m} p_{m-1,j} \right]$.

Since ${}_{(m)}P$ is irreducible for all m , it follows that $X^{(m)}$ constitutes an irreducible Markov chain for all m [6].

The states $\{1, \dots, m-1\}$ form a transient set and m is an absorbing class of states. The n -step transition probability matrix ${}_{(m)}T^n$ can be written as:

$${}^{(m)}I^n = \begin{bmatrix} {}^{(m)}P^n & (I_m + {}^{(m)}P + {}^{(m)}P^2 + \dots + {}^{(m)}P^{n-1})P_m \\ 0 & I \end{bmatrix} \quad (8)$$

Where ${}^{(m)}P^n = ({}^{(m)}p_{ij})^n = p_{ij}(n)$.

The transition probability between realizable classes of states is 1.

This probability also indicates the priorities between the states of the system.

4. Example of pulse width modulation governed by periodic Markov chains

In this example, applicable to railway data transmitter CN-75-6, our goal is to generate a switching function in which blocks of pulses have deterministic duty ratios: [0.75, 0.5, 0.25]. The periodic Markov chain shown in figure 3, with six states divided to three classes, is an example of a solution to such a problem [6], [7]. A short (duration 3/4) and a long (duration 5/4) cycle is available in each of the four classes.

According to the theoretical approach in the previous paragraph, we build a simplified Markov chain in figure 4.

The Markov chains in figure 3, respectively in figure 4 have the same transition probabilities between states S_i , $i=1, \dots, 8$ of classes C_j , $j=1, \dots, 4$. We notice that the Markov chain in figure 4 is more intuitive and tidy than the one in figure 3.

The transition probabilities equal to 1 in figure 4 are conditioned by the existence of transition probabilities between the states of different classes.

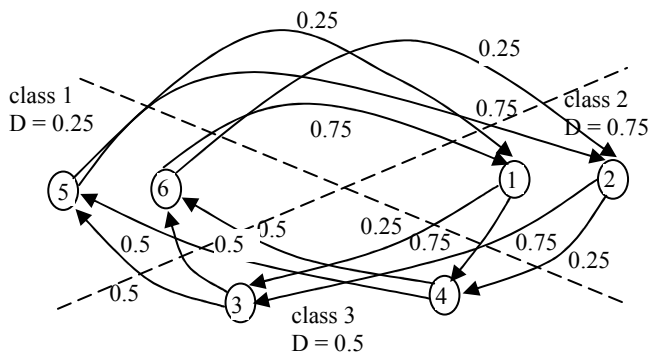


Figure 3. Classic Markov chain for modeling the switching example

The above given duty ratios in the present example and modeled with Markov chains in figure 3, respectively in figure 4, represent the switching between colors red,

yellow, red, green displayed by railway traffic lights commanded by data transmitter CN-75-6.

We analyze this Markov chain with equation (4) and we compare the theoretical predictions with the estimates obtained in Monte Carlo simulations. The agreement between the two is quite satisfactory: the theoretical prediction for the impulse strength at $f = 4$ is 0.0036, and the estimated value is 0.0037.

The Markov chain represented in figure 4 allows dealing with many more classes of states because graphical representation is simplified and can significantly improve the tractability of the optimization of the Markov chains with many states.

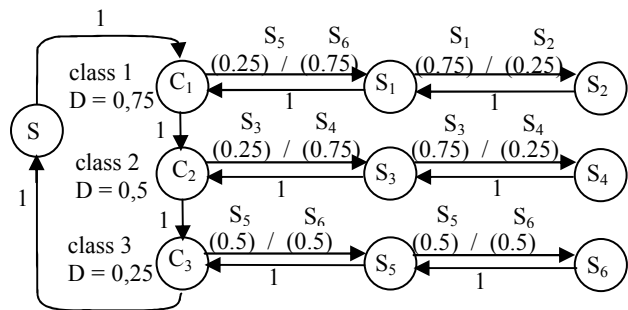


Figure 4. Proposed Markov chain for modeling the switching example

5. Reliability estimation of pulse width modulation with periodic Markov chains

Visits to rare state of failure (F) in recurrent Markov chains are some events. The Markov chain for reliability model has at least three states [7, 8]: starting-state S, working state W, and fail state F. State sequences are realizations of reliability Markov chain model (RMCM). A realization from S to first occurrence of W represents a single successful execution cycle of data transmission. A transition from any state to F represents a failure to data transmission.

The probabilities on arcs in RMCM are the values estimated for the usage profile and component reliabilities expected in practice [9]. We notice that if state $i \in \text{RMCM}$ $i \neq F$, has been visited n_i times and exited without failure, then the probability of failure at state i is no greater than $1/(n_i + 1)$. Let random variable n_F be the number of visits to F in a randomly generated realization of n transitions starting in a state S. Let $\lambda = E(n_F)$ be the mean value of the probability low of n_F . Let $P = [p_{ij}]$ denote the RMCM's transition probability matrix. The RMCM have all states reachable from S by

traces with nonzero probability and arcs from both F and W to S with $P_{FS} = P_{WS} = 1$, so that a successful or unsuccessful path terminated in W, respectively in F state causes an immediate restart in initial state S. RMCM's steady-state probability distribution $\Pi = [\pi_S, \dots, \pi_F]$ is the unique solution of $\Pi = \Pi P$ where $\sum \Pi_i = 1$ and $\pi_i > 0$ is the limiting relative frequency of occurrence of state i as a count transition, e.g., recurrence time, [4].

Adopting a Poisson law with parameter λ , developments in small number laws stress that $P_0(\lambda)$ is an approximation and compute an upper bound for measuring the distance between $P_0(\lambda)$ and the probability law $L(n_F)$.

The total variational distance $d_{TV}[L(n_F), P_0(\lambda)]$ is defined as:

$$d_{TV}[L(n_F), P_0(\lambda)] = \sup_A |L(n_F)(A) - P_0(\lambda)(A)| \quad (9)$$

for events A in the sample space [10], where $P_0(\lambda)$ is the Poisson distribution with parameter λ .

Since Π_F equals the limiting relative frequency of state F, for large n we have:

$$\Pi_F \approx \frac{E(n_F)}{n} = \frac{\lambda}{n} \quad (10)$$

We may approximate for large n, and rare state F:

$$L(n_F) \approx \frac{(n \Pi_F)^k}{k!} \cdot e^{-n \Pi_F} \quad (11)$$

We notice that $\lambda \approx n \cdot \Pi_F$ is the approximate parameter for a full sequence of transition, not per transition. Since the mean sequence length is m_{SS} transitions, the expected count of transitions between visits to F is [4, 11]:

$$m_{SS} E(\lambda) = m_{SS} \frac{\pi_S}{\pi_F} = m_{SS} \frac{m_{FF}}{m_{SS}} = m_{FF} \quad (12)$$

We exemplify this approach on the three state Markov chain (RMCM) given in figure 5. This RMCM corresponds to the Markov chain given in figure 4, where we added the fail state F associated to the states C_i , $i = 1, 2, 3$ associated to the classes in the Markov chain model for pulse width modulation scheme discussed in section 3. By colligating the RMCM in figure 5 with the Markov chain in figure 4, we notice the transition probabilities for the ordinary usage-states given in figure 4 (p_{ij}) and the small probabilities P_{C_iF} , $i = 1, 2, 3$ in figure 5.

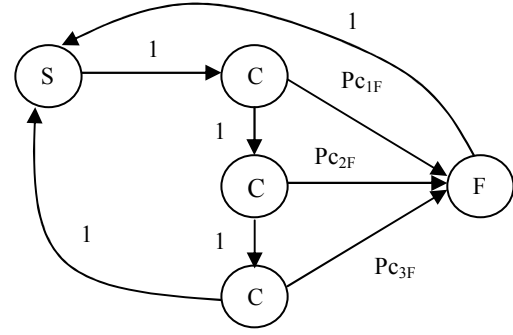


Figure 5. Five state RMCM associated to the Markov chain in figure 4

Given that for highly reliable security communication systems we may have $0 \leq P_{C_iF} \leq 10^{-3}$, and correspondingly $0 < \Pi_{Fi} \leq 1,3 \cdot 10^{-4}$, which allow us to presume that for $P_{C_iF} \approx 10^{-4}$ the stationary distribution vector is:

$$\Pi = [\pi_S, \pi_{C1}, \pi_{C2}, \pi_{C3}, \pi_F] = [0.14278, 0.42813, 0.14283, 0.14276, 1.42689 \cdot 10^{-5}] \quad (13)$$

The vector of mean recurrence time is:

$$[m_{SS}, \dots, m_{FF}] = \left[\frac{1}{\pi_S}, \dots, \frac{1}{\pi_F} \right] = [7.00400, 2.33412, 7.00120, 7.00470, 7.00821 \cdot 10^4] \quad (14)$$

The vector of the expected number of occurrences of states between transitions to non-rare state S is:

$$\frac{\pi}{\pi_S} = [1, 3.0009, 1.0005, 1.0005, 0.9989, 0.0005] \quad (15)$$

The vector of the expected number of occurrences of states between transitions to rare state F is:

$$\frac{\pi}{\pi_F} = [1.0009, 3.0047, 1.0009, 1.0012, 0.0003] \cdot 10^4 \quad (16)$$

The mean recurrence time of state S is $m_{SS} \approx 9$; therefore $n=9 \cdot 10^4$ transitions correspond to approximately 10^4 average sequences from S to S. The Poisson approximation of $L(n_F)$ (see relation (11) for $n = 9 \cdot 10^4$ and $\lambda = 9\pi_F \cdot 10^4$ stands $p_{CF} = 10^{-4}$ and the rare state F has steady-state probability $\pi_F \approx 1.43 \cdot 10^{-5}$, and

the upper bound $0,903 \cdot 10^{-5}$, where $k = 0,1,2,3$. The MTTF is $m_{FF} \approx 9 \cdot 10^4$ for $p_{CF} = 10^{-4}$.

6. Conclusions

Our paper focuses syntheses results for randomized modulation strategies achieved with Markov chains, suitable for power converter, for example the one used in the railway data transmitter CN-75-6.

Randomized modulation switching schemes governed by Markov chains applicable to d.c./a.c. or d.c./d.c. converters have been described.

We notice that most of the previous results in which randomizations of the switching pulse train are statistically independent and governed by invariant probabilistic rules.

While these implementations tend to be very successful in achieving certain kinds of spectral shaping in frequency domain, they fail in short offering time-domain performance guarantees concerning the switching process.

This is objectionable in many cases, for example when accumulated deviations of the randomized switching waveform from the nominal waveform determine errors in data transmission.

This problem, together with the lack of a widely known and accepted analysis framework for randomized switching waveforms, impediment the wider use of randomized modulation.

In this paper we describe a class of stationary randomized modulation schemes that allow control of the time domain performance of randomized switching, together with the spectral shaping in the frequency domain. In order to do this, the switching signal comprises a concatenation of distinct waveforms segments chosen in sequence according to a Markov chain model.

Our representation for complex periodic Markov chains we believed to be novel.

We also believe that this new model offers a new perspective for the spectral characteristics and other associated waveforms in a converter to the probabilistic structure that governs the dithering of an underlying deterministic nominal switching pattern.

Further research will continue to focus on minimization of one or multiple discrete harmonics. This approach corresponds to cases where the narrow-band characteristics corresponding to discrete harmonics are harmful, as for example in the railway traffic security.

We also discussed results in rare events for security communication system based on finite-state, discrete-

parameter, recurrent Markov chain, here entitled reliability Markov chain model (RMCM). The chain provides a simple definition of failure as a rare event, respectively as a failure state F for which the steady-state Π_F is orders of magnitude smaller than Π_K for $k \neq F$, usually states of RMCM.

Poisson law distribution bounds the transitions to a rare-fail state F in arbitrarily large size RMCM.

Further research will focus on improvement of the analytic capabilities of RMCM in the study of extreme values of rare events [21-25] when failure is infrequent and MTTF is long.

10. Acknowledgements

This work is part of the Grant CEEX No. 5954/18.09.2006 financed by the Romanian Ministry of Education and Research.

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