

Performance Evaluation of Distributed Systems

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Abstract— Some evaluation techniques for distributed systems are prevented. In order to model clearly the synchronization, involved in these systems, a Petri net model is used. We focus on the performance evaluation of a strongly connected event graph with random firing times. We have an upper bound and a lower bound for the average cycle time of event graphs knowing the initial marking. We propose an algorithm to evaluate the bounds used to calculate an average cycle time. An application of the results to the evaluation of a Kanban system is proposed.

Keywords— Cycle-time, Petri net, distributed system, upper bound, lower bound, firing time.

I. INTRODUCTION

IN this paper, we focus on techniques for the prediction and the verification of performance of distributed systems. We consider a distributed system as a loosely or a tightly coupled processing elements working co-operatively and concurrently on a set of related tasks. In general, there are two approaches for performance evaluation [1]: deterministic models and probabilistic models. In deterministic models, it is usually assumed that the task arrival times, the task execution times, and the synchronisation involved are known in advance to the analysis. This approach is very useful for performance evaluation of real-time control systems with hard deadline requirements. In probabilistic models, the task arrival rates and the task service time are usually specified by probabilistic distribution functions. Probabilistic models usually give a gross prediction on the performance of a system and are good for easy stages of system design when the system characteristics are not well understood. In this paper, we focus on performance analysis of distributed systems and for to model clearly the synchronisation

involved in concurrent systems, the Petri net model is chosen. In this paper we consider event graphs as Petri nets in which each place has one input transition and one output transition. It has been shown that distributed systems can be modelled as event graphs [2], [3]. When the manufacturing times are deterministic (respectively stochastic), the cycle time (respectively the mean cycle time) of the model is the period (respectively the mean period) required to manufacture a given set of parts which fits with the required ratios. The smaller the cycle time (respectively the mean cycle time) the higher the productivity of the system.

When the firing times of transitions are deterministic, it is possible to define the cycle time of an elementary circuit. This is given by the ratio of the sum of the firing times associated with the transitions of the circuit by the number of its tokens, which is constant (we address strongly connected graphs, which have the number of tokens in any elementary circuit, constant):

$$C_i = \frac{T_i}{N_i}, i = 1, 2, \dots, n \quad (1)$$

Where i = number of elementary circuits of the graph;
 C_i = cycle time of elementary circuits i ;

$T_i = \sum_i^n r_i$ is the sum of the execution times of the transition in circuit i ;

$N_i = \sum_i^n M_i$ is the total number of tokens, in the places in circuit i ;

In this case it has been proven that the cycle time of a strongly connected event graph is equal to the greatest cycle time of all elementary circuits. Furthermore, given a value C^* greater than the largest firing time of all transitions, an algorithm has been proposed in [2] to reach a cycle time less than C^* , while minimizing a linear combination of the number of tokens in the places. The coefficients of the linear combination are the elements of a p-invariant. When the event graph is the model of a ratio-driven distributed system (such as manufacturing system), C^* has to be greater than the largest cycle time of all command circuits [4].

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A command circuit is an elementary circuit, which joins the transition which model the operations performed on the same machine. Such a circuit contains one token to prevent more than one transition firing at any time in each elementary circuit.

In other words, C^* must be greater than the time required by the bottleneck machine to perform a sequence of parts which fits with the production ratios.

In the case of random firing times, it is no longer possible to take advantage of the elementary circuits to evaluate the behavior of the event graph and to reach a given performance. Thus, the results presented in this paper, which aim at reaching a given mean cycle time in a steady state while minimizing a linear combination of the place markings, are particularly important at the preliminary design level of manufacturing systems working on a ratio-driven basis. This applies in particular to distributed systems in flexible manufacturing systems.

II. FRAMING THE MEAN CYCLE TIME

It has been proven [5] that a marking belonging to the optimal solution under a periodic operational mode (POM) is an optimal solution under an earliest operational mode (EOM). So, we consider the earliest operational mode of the event graph, and we assume only non pre-emptive transitions firings. We further assume that, when transition fires, the related tokens remain in the input places until the firing process ends. They then disappear, and one new token appears in each output place of the transition.

We use the following notations:

M_i = the marking of the elementary circuits, $i \in N$.

$X_t^k \in \mathbf{R}^+$ = random variable generating the time required for the k^{th} firing of transition t , $k \in N$.

$I_t(n)$ = instant of the n^{th} firing initiation of transition t ;

E = set of elementary circuits;

$s(e)$ = sum of the random variables generating the firing;

$\sum_{t \in e} X_t^1$ = sum of times of the transitions belonging to e ;

E_t = set of elementary circuits containing transition t .

We assume that the sequences of transition firing times are independent sequences of integrable random variables. It was proven in [4] that there exists a positive constant $s(M_0)$ such that:

$$\lim_{n \rightarrow \infty} \frac{I_t(n)}{n} = C_m \quad (2)$$

Where:

C_m = the average cycle time of the event graph. Furthermore, we denote by m_t the mean value of X_t^k and

by q_t the standard deviation of X_t^k , i.e., $m_t = F[X_t^k]$ and

$$q_t^2 = F[(X_t^k - m_t)^2].$$

A. The lower bound of mean cycle time

The cycle time of the deterministic problem obtained by replacing the random variables, which generate the firing times, by their mean values is a lower bound of the mean cycle time [4]. The following relation proven in [2] provides a better lower bound for the value of the mean cycle time than the previous one:

$$C^{**} \geq \max_{e \in E} F[\max\{\frac{s[e \setminus \{t^*(e)\} + m_{t^*(e)}]}{M_0(e)}, m_{t^*(e)}\}] \quad (3)$$

Where $t^*(e)$ is a transition with the greatest average firing time, i.e., $m_{t^*(e)} = \max_{t \in e} m_t$.

B. The upper bound of mean cycle time

With M_0 being initial marking, we derive a marking M_1 from M_0 by leaving the places, which are empty in M_0 , empty in M_1 and by reducing to one the number of tokens in the places containing more than one token in M_0 .

Thus, $M_1(p) \leq M_0(p)$ for any set of places of the strongly connected event graph. An earliest operation mode running with the initial marking M_1 leads to a greater mean cycle time than the one obtained when starting from M_0 . Then, starting from M_1 , we apply to the event graph the earliest operation mode, but we block the tokens as soon as they reach a place already marked in M_1 . This operation mode is referred as the constrained mode [3].

We denote by C^* the mean cycle time obtained by using the constrained operation mode when M_1 is the initial marking. We know [2] that C^* is greater than the mean cycle time obtained by using the earliest operation mode starting from M_1 which, in turn, is greater than the mean cycle time obtained with the earliest operation mode when the initial marking is M_0 . Thus, C^* is an upper bound of the solution to our problem (i.e. the mean cycle time obtained starting from M_0 when using the earliest operation mode). The following relation defines this upper bound:

$$C^* = F[\max_{z \in Z} s(z)] \quad (4)$$

Where Z is the set of directed path verifying the following properties:

- the origin and the extremity of any path is a marked place;
- it is no marked place between the origin and the extremity of the path.

III. EVALUATION OF THE EVENT GRAPH

In the reminder of the previous section, we compare the previous bounds with the existing ones. Under the assumption of non-preemptive transition firing, it was proven in [1] that:

$$C^{**} \geq \max_{e \in E} F[\max\{s[e \setminus \{t^*(e)\} + m_{t^*(e)}], m_{t^*(e)}\}] =$$

= old lower bound (5)

$$C^* \leq \sum_t m_t = \text{old upper bound.} \quad (6)$$

The following relations show that the new bounds are better than the old ones. But how closer are they to the optima solution? In order to answer this question we give the next algorithm, inspired from operational research area, for verifying system performance:

- Express the token loading in a $p \times p$ matrix P , where p is the number of places in the Petri net model of the system. Entry (A,B) in the matrix equals x if there are x tokens in place A and place A is connected directly to place B by a transition; otherwise (A,B) equals 0.
- Express transition time in an $p \times p$ matrix Q . Entry (A,B) in the matrix equals to the mean values of the random variables which generate the firing times (i.e., X_t^k) if A is an input place of transition i and B is its output place. Entry (A,B) contains the symbol "w" if A and B are not connected directly as described above.
- Compute matrix $CP-Q$ (with $p - w = \infty$, and $C = (C^* + C^{**})/2$, for $p \in \mathbb{N}$), then use Floyd's algorithm to compute the shortest distance between every pair of nodes using matrix $CP-Q$ as the distance matrix. The result is stored in matrix S . There are three cases:

- All diagonal entries of matrix S are positive (i.e., $CN_k - T_k > 0$ for all circuits - see relation (1)) the system performance is higher than the given requirement;
- Some diagonal entries of matrix S are zero's and the rest are positive (i.e., $CN_k - T_k = 0$ for some circuits and $CN_k - T_k > 0$ for the other circuits) - the system performance just meet the given requirement;
- Some diagonal entries of matrix S are negative (i.e., $CN_k - T_k < 0$ for some circuits) - the system performance is lower than the given requirement.

In addition we may say that when a decision-free system runs at its highest speed, CN_k equals to T_k for the bottleneck circuit. This implies that the places in the bottleneck circuit will have zero diagonal entries in matrix S . System performance can be improved by reducing the execution times of some transitions in the circuit, or introducing more concurrency in the circuit (by modifying the initial marking), or increasing the mean cycle time (by choosing another average value for this).

IV. EVALUATION OF KANBAN SYSTEM OPERATIONS

As we well know [3], an event graph can be used to model a Kanban system. An example of simple production line will be use to exemplify the above discussed problems. The production line consists of two machining tools (M_1 and M_2), two robot arms and two conveyors. Each machining tool is serviced by a dedicated robot arm, which performs load and unload tasks. One conveyor is used to transport workpieces, a maximum of two at a time. The other conveyor is used to transport empty pallets. There are three pallets available in the system. Each workpiece is machined on M_1 and M_2 , in this order. The stochastic timed Petri net model of this system is shown in Fig.1. The initial marking of the net is $(300012111)^T$.

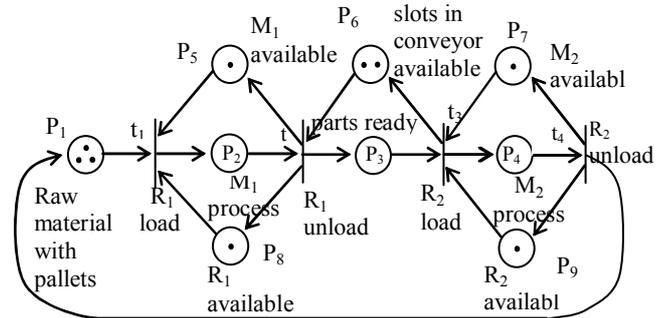


Fig.1. Stochastic Petri net model for a manufacturing system

When time delays are modeled as random variables, it has become a convention to associate time delays with the transition only. The transition involved, have the associated time delays expressed in time units. The random variables X_1, X_2, X_3, X_4 are assigned to the transitions t_1, t_2, t_3, t_4 , respectively. X_1 is uniformly distributed on $[0,2]$, X_2 and X_3 are random variables with $F[X_2] = 11$ t.u. and $F[X_3] = 1$ t.u. X_4 is a constant and equal to 17 t.u. The Petri net model contains four loops. The time delays associated with these loops, as well as their token contents are:

- 1) loop: $t_1 p_2 t_2 p_3 t_3 p_4 t_4 p_1 t_1$, loop delay: 30 t.u., token sum: 3, cycle time: 10 t.u.

- 2) loop: $t_1 p_2 t_2 p_5$ (or p_8) t_1 , loop delay: 12 t.u., token sum: 1, cycle time: 12 t.u.
- 3) loop: $t_2 p_3 t_3 p_6 t_2$, loop delay: 2 t.u., token sum: 2, cycle time: 1 t.u.
- 4) loop: $t_3 p_4 t_4 p_7$ (or p_9) t_3 , loop delay: 18 t.u., token sum: 1, cycle time: 18 t.u.

Then, the minimum cycle time is 18 time units. This means that it takes a minimum of 18 time units to transform a raw workpiece into a final product. Computing the lower-bound and the upper bound of cycle time of the event graph given in Fig.1., using the relations (3) and (4), we obtain the values: $C^{**} \geq 12$ t.u.; $C^* \leq 18$ t.u.

CONCLUSIONS

An important result in this paper is that it is always possible to reach a mean cycle time as close as possible to the greatest mean firing time using a finite marking, assuming that a transition cannot be fired by more than one token at each time. This result holds for any distribution of the transition firing time.

An algorithm for verifying the distributed systems performance was introduced. An approach for computing upper and lower bounds of the performance of a conservative general system is presented. However, the bounds produced may be loose.

Further research will focus the condition under which a mean cycle time can be reached with a finite marking.

REFERENCES

- [1] F. Bacelli and Z. Lin, "Compression properties of stochastic decision free Petri nets", *IEEE Trans. Autom. Contr.*, vol 37, no. 12, pp. 1905-1920, 1992.
- [2] S. Laftit, J. M. Proth and X.L.Xie, "Optimization of invariant criteria for event graphs", *IEEE Trans. Autom. Contr.*, vol 37, no. 12, pp. 547-555, 1992.
- [3] J. M. Proth and X.L.Xie, "Cycle time of stochastic event graphs: evaluation and marking optimization", *IEEE Trans. Autom. Contr.*, vol 39, no. 7, pp. 1482-1486, 1994.
- [4] J. Campos, G. Chiola and M. Silva, "Properties and performance bounds for closed free choice synchronized monoclase queuing networks", *IEEE Trans. Autom. Contr.*, vol 36, no. 12, pp. 1368-1382, 1991.
- [5] R. Zurawski and M. C. Zhon, "Petri nets and industrial applications: a tutorial", *IEEE Trans. Ind. Electr.*, vol 41, no. 6, pp. 567-583, 1994.
- [6] J. Singh, Operations Research, Penguin Books, Middlessex, England, 1999.
- [7] N. Viswanadham and R. Ram, "Composite performance dependability analysis of cellular manufacturing systems", *IEEE Trans. on Rob. and Autom.*, vol. 12, pp. 245-258, 1994.
- [8] R. Zurawski and M. C. Zhon, "Petri nets and industrial applications: a tutorial", *IEEE Trans. on Ind. Electronics*, vol. 41, no 6, pp. 567-583, 1994.
- [9] G. Ciardo, J. Muppala and K. S. Trivedi, "On the solution of GSPN reward models", *Performance Evaluation*, vol. 12, pp. 237-253, 1991.
- [10] T. Murata, "Petri Nets: Properties, analysis and applications", *Proc. IEEE*, vol. 77, pp.541-580, 1989.
- [11] C. Ciufudean, "Petri nets in railway traffic systems", *Advances in electrical and computer engineering, University "Stefan cel Mare" of Suceava*, vol 1(18), no 1(15), pp.21-26, 2001.
- [12] C. Ciufudean, Discrete event systems for modeling the railway traffic, Matrix Rom, Bucuresti, Romania, 2002.
- [13] C. Ciufudean, A.B. Larionescu, "Safety criteria for production lines modeled with Petri nets", *Advances in Electrical and Computer Engineering*, vol. 2(9), no.2(18), pp.15-20, 2002.
- [14] J.Ezpeleta, J.M. Colom, J. Martinez, "A Petri net based deadlock prevention policy for flexible manufacturing systems", *IEEE Trans. on Rob. and Autom.*, vol.11, no2., pp.1028-1036, 1996.